

Type checking rules

The following rules applies to my whole type checking rules:

1. " σ " is a state.
2. " $e1$ ", " $e2$ ", ..., " e_n " are expressions.
3. " $c1$ ", " $c2$ " are commands.
4. " t ", " $t1$ ", " $t2$ ", ..., " t_n " are types.
5. " x " is a variable.

Type checking of numerical arithmetic:

$$\frac{\langle e1, \sigma \rangle \Downarrow i32, \langle e2, \sigma \rangle \Downarrow i32}{\langle e1 + e2, \sigma \rangle \Downarrow i32} \quad (1)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow i32, \langle e2, \sigma \rangle \Downarrow i32}{\langle e1 * e2, \sigma \rangle \Downarrow i32} \quad (2)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow i32, \langle e2, \sigma \rangle \Downarrow i32}{\langle e1 - e2, \sigma \rangle \Downarrow i32} \quad (3)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow i32, \langle e2, \sigma \rangle \Downarrow i32}{\langle e1 / e2, \sigma \rangle \Downarrow i32} \quad (4)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow i32, \langle e2, \sigma \rangle \Downarrow i32}{\langle e1 \% e2, \sigma \rangle \Downarrow i32} \quad (5)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow i32, \langle e2, \sigma \rangle \Downarrow i32}{\langle e1 \wedge e2, \sigma \rangle \Downarrow i32} \quad (6)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow i32}{\langle -e1, \sigma \rangle \Downarrow i32} \quad (7)$$

Type checking of boolean arithmetic:

$$\frac{\langle e1, \sigma \rangle \Downarrow bool, \langle e2, \sigma \rangle \Downarrow bool}{\langle e1 \&\& e2, \sigma \rangle \Downarrow bool} \quad (8)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow bool, \langle e2, \sigma \rangle \Downarrow bool}{\langle e1 \parallel e2, \sigma \rangle \Downarrow bool} \quad (9)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow bool}{\langle !e1, \sigma \rangle \Downarrow bool} \quad (10)$$

Type checking of other operations:

equals

$$\frac{\langle e1, \sigma \rangle \Downarrow bool, \langle e2, \sigma \rangle \Downarrow bool}{\langle e1 == e2, \sigma \rangle \Downarrow bool} \quad (11)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow i32, \langle e2, \sigma \rangle \Downarrow i32}{\langle e1 == e2, \sigma \rangle \Downarrow bool} \quad (12)$$

not equals

$$\frac{\langle e1, \sigma \rangle \Downarrow bool, \langle e2, \sigma \rangle \Downarrow bool}{\langle e1 != e2, \sigma \rangle \Downarrow bool} \quad (13)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow i32, \langle e2, \sigma \rangle \Downarrow i32}{\langle e1 != e2, \sigma \rangle \Downarrow bool} \quad (14)$$

less than

$$\frac{\langle e1, \sigma \rangle \Downarrow i32, \langle e2, \sigma \rangle \Downarrow i32}{\langle e1 < e2, \sigma \rangle \Downarrow bool} \quad (15)$$

parentheses

$$\frac{\langle e1, \sigma \rangle \Downarrow i32}{\langle (e1), \sigma \rangle \Downarrow i32} \quad (16)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow bool}{\langle (e1), \sigma \rangle \Downarrow bool} \quad (17)$$

function call

$$\frac{\langle e1, \sigma \rangle \Downarrow t1, \langle e2, \sigma \rangle \Downarrow t2, \dots, \langle en, \sigma \rangle \Downarrow tn, \langle f(t1, t2, \dots, tn), \sigma \rangle \Downarrow i32}{\langle f(e1, e2, \dots, en), \sigma \rangle \Downarrow i32} \quad (18)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow t1, \langle e2, \sigma \rangle \Downarrow t2, \dots, \langle en, \sigma \rangle \Downarrow tn, \langle f(t1, t2, \dots, tn), \sigma \rangle \Downarrow bool}{\langle f(e1, e2, \dots, en), \sigma \rangle \Downarrow bool} \quad (19)$$

$$\frac{\langle e1, \sigma \rangle \Downarrow t1, \langle e2, \sigma \rangle \Downarrow t2, \dots, \langle en, \sigma \rangle \Downarrow tn, \langle f(t1, t2, \dots, tn), \sigma \rangle \Downarrow \emptyset}{\langle f(e1, e2, \dots, en), \sigma \rangle \Downarrow \emptyset} \quad (20)$$

Commands

if

$$\frac{\langle c1, \sigma \rangle \Downarrow i32, \langle c2, \sigma \rangle \Downarrow i32}{\langle \text{if } e1 \text{ then } c1 \text{ else } c2, \sigma \rangle \Downarrow i32} \quad (21)$$

$$\frac{\langle c1, \sigma \rangle \Downarrow \text{bool}, \langle c2, \sigma \rangle \Downarrow \text{bool}}{\langle \text{if } e1 \text{ then } c1 \text{ else } c2, \sigma \rangle \Downarrow \text{bool}} \quad (22)$$

$$\frac{\langle c1, \sigma \rangle \Downarrow \emptyset, \langle c2, \sigma \rangle \Downarrow \emptyset}{\langle \text{if } e1 \text{ then } c1 \text{ else } c2, \sigma \rangle \Downarrow \emptyset} \quad (23)$$

$$\frac{\langle c1, \sigma \rangle \Downarrow \emptyset}{\langle \text{if } e1 \text{ then } c1, \sigma \rangle \Downarrow \emptyset} \quad (24)$$

while

$$\frac{}{\langle \text{while } e1 \text{ do } c1, \sigma \rangle \Downarrow \emptyset} \quad (25)$$

declaration, assignment

$$\frac{}{\langle x := n, \sigma \rangle \Downarrow \emptyset} \quad (26)$$